Event Studies with Feedback*

Irene Botosaru
McMaster University

Laura Liu
University of Pittsburgh

January 8, 2026

Abstract

Event studies often conflate direct treatment effects with indirect effects operating through endogenous covariate adjustment. We develop a dynamic panel event study framework that separates these effects. The framework allows for persistent outcomes and treatment effects and for covariates that respond to past outcomes and treatment exposure. Under sequential exogeneity and homogeneous feedback, we establish point identification of common parameters governing outcome and treatment effect dynamics, the distribution of heterogeneous treatment effects, and the covariate feedback process. We propose an algorithm for dynamic decomposition that enables researchers to assess the relative importance of each effect in driving treatment effect dynamics.

Keywords: Event study, heterogeneous treatment effects, dynamic panel data, sequential exogeneity, feedback mechanisms
JEL classification: C23, C21

1 Introduction

Event studies in the panel data setting aim to quantify how treatment effects evolve over time and vary across units. When outcomes are persistent and covariates adjust endogenously to past outcomes and treatment exposure, observed dynamic responses may not correspond to a single causal mechanism. Instead, they may combine direct effects with indirect effects operating through endogenous covariate adjustments. This distinction is empirically relevant when treatment triggers equilibrium adjustment. For example, a minimum wage policy may have a direct effect on wages but may also lead to changes in firm-level input demand, which may in turn affect individual wages. Separating these effects permits an assessment of how

*Botosaru: Department of Economics, McMaster University, email: `botosari@mcmaster.ca`. Liu: Department of Economics, University of Pittsburgh, email: `laura.liu@pitt.edu`. We thank Jeffrey Wooldridge and participants at the 2026 AEA annual meeting for helpful discussions. Botosaru gratefully acknowledges financial support from the Canada Research Chairs Program. All remaining errors are our own.

much of the dynamic response reflects direct versus indirect effects through equilibrium-induced covariate adjustment.

This paper develops a dynamic event study framework that separates these two effects. The *direct structural effect* captures how outcomes respond to treatment holding the covariate path fixed, while the *indirect adjustment effect* operates through covariates that evolve endogenously over time. We consider a panel event study setting with units $i = 1, \ldots, N$ observed over periods $t = 0, \ldots, T$. For exposition, we present a simplified version of the model. The outcome evolves in calendar time t according to

$$Y_{it} = \rho_Y Y_{i,t-1} + \alpha_i + X_{it}'\beta + \sum_{j \in \mathcal{J}} D_{it}^j \delta_{ij} + U_{it}, \quad U_{it} \overset{iid}{\sim} \mathcal{N}(0, \sigma_U^2), \quad t = 1, \ldots, T, \tag{1}$$

where $Y_{it} \in \mathbb{R}$ is the scalar outcome, $X_{it} \in \mathbb{R}^K$ is a vector of time-varying covariates, and α_i captures unit-specific heterogeneity. Treatment is characterized by an event-time $t_{0i} \in \{1, \ldots, T\}$. Let $\mathcal{J} \subset \mathbb{Z}$ denote a finite set of event-time indices for which dynamic treatment effects are specified. For each $j \in \mathcal{J}$, the event-time indicator $D_{it}^j = \mathbb{1}\{t - t_{0i} = j\}$ is a deterministic function of t_{0i}. The coefficients δ_{ij} represent unit-specific dynamic treatment effects at event time j and capture the direct structural response to treatment.

Following Botosaru and Liu (2025), we impose a parsimonious dynamic structure on these heterogeneous treatment effects by assuming that they follow an autoregressive process in event time:

$$\delta_{ij} = \rho_\delta \delta_{i,j-1} + \varepsilon_{ij}, \quad \varepsilon_{ij} \overset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2), \quad j = 1, \ldots, J, \tag{2}$$

where ρ_δ is a common persistence parameter and ε_{ij} are idiosyncratic innovations. Let $\lambda_i = (\alpha_i, \delta_{i0})'$ denote the vector of unobserved heterogeneity, and denote its conditional distribution by

$$H(\lambda_i \mid \mathcal{I}_i^0), \quad \mathcal{I}_i^0 = \{Y_{i0}, X_{i0}, t_{0i}\}.$$

The framework readily extends to higher-order dynamics, continuous treatment intensities, and event-time leads for testing anticipation effects. Gaussianity in (1) and (2) is imposed for likelihood-based estimation, while the identification results do not require a correct Gaussian specification.

The parameter β captures the indirect adjustment channel, mapping changes in covariates into changes in outcomes. In many applications, these covariates are policy-reactive and adjust dynamically in response to treatment and past outcomes. As a result, treatment may affect outcomes not only directly, but also indirectly through sequentially exogenous covariate adjustments. Standard event study approaches typically rule out this feedback by imposing strict exogeneity of covariates. While Botosaru and Liu (2025) relax this requirement to allow for sequential exogeneity, they treat the covariate process as given and do not model or identify the adjustment mechanism itself.

This paper extends their framework by explicitly modeling and identifying the covariate feedback process. We allow covariates to be predetermined and impose a *homogeneous feedback* restriction, requiring the covariate adjustment rule to be common across units up to observable conditioning variables. This restriction separates the two channels: the distribution $H(\lambda_i \mid \mathcal{I}_i^0)$ governs unit-level heterogeneity in the direct structural treatment effect, while β and the identified feedback process capture the indirect adjustment effects. The framework therefore permits the inclusion of policy-reactive covariates and delivers an explicit decomposition of event study dynamics beyond reduced-form comparisons, facilitating policy analysis in settings with sequentially exogenous covariate adjustment.

2 Model and Assumptions

We denote the history of variables up to time t as $Y_i^t = (Y_{i1}, \ldots, Y_{it})'$ and $X_i^t = (X_{i1}', \ldots, X_{it}')'$. Let $\theta = (\rho_Y, \rho_\delta, \beta, \sigma_U^2, \sigma_\varepsilon^2)'$ denote the vector of common structural parameters, and $\mathcal{I}_i^t = \left\{Y_i^t, X_i^t, \mathcal{I}_i^0\right\}$ denote the information set up to time t.

Assumption 1 (Sequential Exogeneity). *For all $t = 1, \ldots, T$, conditional on $(\mathcal{I}_i^{t-1}, X_{it}, \lambda_i)$, Y_{it} does not depend on future outcomes, covariates, or shocks; conditional on $(\mathcal{I}_i^{t-1}, \lambda_i)$, X_{it} does not depend on current shocks, nor on future outcomes, covariates, or shocks. Moreover,*

$$\mathbb{E}[U_{it} \mid \mathcal{I}_i^{t-1}, X_{it}, \lambda_i] = 0.$$

To achieve point identification of the feedback process in short panels, we impose the following restriction on the feedback process, following Bonhomme (2025).

Assumption 2 (Homogeneous Feedback). *Let $f_t(\cdot \mid \cdot)$ denote the conditional density of X_{it}. For all $t = 1, \ldots, T$,*

$$f_t(X_{it} \mid \mathcal{I}_i^{t-1}, \lambda_i) = f_t(X_{it} \mid \mathcal{I}_i^{t-1}) \quad a.s., \tag{3}$$

with the feedback factor defined below being positive on the support of $\mathcal{I}_i^T$:

$$g(X_i^T \mid Y_i^T, \mathcal{I}_i^0) = \prod_{t=1}^{T} f_t(X_{it} \mid \mathcal{I}_i^{t-1}). \tag{4}$$

Under Assumption 1, covariates X_{it} can depend on λ_i through past outcomes in complex ways. Assumption 2 breaks this dependence by requiring the conditional covariate adjustment rule (3) to be the same across individuals, regardless of their λ_i, conditional on the observable history $\mathcal{I}_i^{t-1}$. This restriction delivers a clean separation between the direct structural effect and the indirect adjustment effect. Conditional on $(X_i^T, \mathcal{I}_i^0)$, the likelihood kernel for Y_i^T given

λ_i coincides with the representation studied in Botosaru and Liu (2025), while all information about the indirect adjustment effect is captured by $g(X_i^T \mid Y_i^T, \mathcal{I}_i^0)$. This factorization allows us to treat the feedback mechanism as a separately identified component and to apply the identification arguments of Botosaru and Liu (2025) to the conditional distribution of Y_i^T given $(X_i^T, \mathcal{I}_i^0)$ to identify the direct structural effect. The positivity condition on $g(X_i^T \mid Y_i^T, \mathcal{I}_i^0)$ ensures that this decomposition is well-defined.

3 Identification

We now establish that the model delivers point identification of both the direct and indirect effects. Specifically, we identify: (i) the common parameters θ, (ii) the distribution of unobserved heterogeneity $H(\lambda_i \mid \mathcal{I}_i^0)$, which characterizes the direct structural effect, and (iii) the feedback process $f_t(X_{it} \mid \mathcal{I}_i^{t-1})$, which characterizes the indirect adjustment effect.

Theorem 1 (Identification). *Suppose* $\left\{Y_{it}, X_{it}, \{D_{it}^j\}_{j\in\mathcal{J}}\right\}_{t=1}^T$ *follow* (1) *and* (2). *Let Assumptions 1–2 hold, and suppose the regularity conditions of Botosaru and Liu (2025) for identification are satisfied (i.i.d. sampling over i, conditional independence of errors over calendar and event times, nonvanishing and differentiable characteristic functions, and rank condition for the event study design). Then,* θ, $H(\lambda_i \mid \mathcal{I}_i^0)$, *and* $\{f_t(X_{it} \mid \mathcal{I}_i^{t-1})\}_{t=1}^T$ *are identified. Moreover, the cohort-specific distribution* $H(\lambda_i \mid t_{0i})$ *and unconditional distribution* $H(\lambda_i)$ *are identified.*

Remark 1 (Time Effects). *It is conventional to allow for additive time effects* $\{\gamma_t\}$ *in the outcome equation. Lemma 1 in the Supplemental Appendix shows that such effects potentially entering* (1) *can be removed by cross-sectional demeaning. Since this transformation leaves* θ *and* λ_i *unchanged, Theorem 1 continues to apply provided Assumptions 1 and 2 are interpreted for the demeaned process* $\left\{\dot{Y}_{it}, \dot{X}_{it}\right\}$ *and the corresponding initial conditions in* $\dot{\mathcal{I}}_i^0$. *Thus* $\{\gamma_t\}$ *are pure nuisance parameters that do not affect identification of* $H(\lambda_i \mid \dot{\mathcal{I}}_i^0)$.

4 Counterfactual Analysis and Dynamic Decomposition

The main contribution of our framework is that it enables the decomposition of dynamic treatment effects into direct and indirect effects, which allows counterfactual analysis incorporating both channels. The identified objects θ, $H(\lambda_i \mid \mathcal{I}_i^0)$, and $\{f_t(X_{it} \mid \mathcal{I}_i^{t-1})\}_{t=1}^T$ characterize both the heterogeneous treatment effects and the dynamic adjustment of covariates following treatment.

The factorization in the proof of Theorem 1 implies that in the first step, the estimation of θ and $H(\lambda_i \mid \mathcal{I}_i^0)$ can proceed as in Botosaru and Liu (2025). In a second step, the homogeneous feedback process for covariates can be estimated by modeling the transition densities $f_t(X_{it} \mid \mathcal{I}_i^{t-1})$, using, e.g., parametric Markov models or sieve methods. Under our assumptions, the parameters governing the direct channel (θ, H) and the feedback mechanism enter the likelihood in separable blocks, so estimation of (θ, H) need not rely on a particular parametric specification of the feedback model, provided the feedback factor g is estimated consistently on the relevant support. The feedback model can therefore be selected to balance flexibility and parsimony depending on the intended decomposition and counterfactual exercises.

Given an alternative treatment timing t_{0i}^* and/or alternative initial conditions $\{Y_{i0}^*, X_{i0}^*\}$, one can construct joint counterfactual paths $\left\{Y_i^{T,*}, X_i^{T,*}\right\}$ as described in Algorithm 1. Starting from the counterfactual initial conditions $\mathcal{I}_i^{0,*}$, latent heterogeneity λ_i is drawn from $H(\lambda_i \mid \mathcal{I}_i^{0,*})$ and counterfactual treatment effects are generated according to (2). The system is then iterated forward in calendar time, drawing covariates from the estimated feedback process and constructing outcomes using (1), which combines the direct effect through $\{\delta_{ij}^*\}$ and the indirect effect through X_{it}^* and β.

The resulting counterfactual paths decompose dynamic event study responses into a direct structural component, driven by latent heterogeneity and treatment effect dynamics, and an indirect component operating through sequentially exogenous covariate adjustments. This decomposition is empirically relevant because it distinguishes between treatment effects that arise from heterogeneous structural responses versus those that operate through equilibrium adjustments. For example, in a minimum wage study, the direct effect captures how firm productivity responds to the wage change, while the indirect effect captures how firm adjustments in hours or employment composition feed back into productivity. The framework provides an approach to quantifying each effect, enabling researchers to assess their relative importance in driving event study dynamics and facilitating counterfactual analysis that propagates both effects.

5 Extensions

First, the homogeneous feedback framework is not tied to the linear model in (1). In principle, one can replace the outcome equation by a nonlinear panel model $Y_{it} \sim f_\theta(\cdot \mid \mathcal{I}_i^{t-1}, X_{it}, \lambda_i)$, where θ is a finite-dimensional parameter and λ_i collects unobserved unit-specific heterogeneity. Examples include dynamic logit or probit models, Poisson or negative binomial count models, and Tobit models for censored outcomes. Under Assumptions 1–2, once we factor out the feedback term, the conditional distribution of $Y_i^T \mid X_i^T, \mathcal{I}_i^0$ depends on (θ, H) only through an average over λ_i. Identification of (θ, H) can then proceed by applying the relevant nonlinear identification results to this conditional distribution, while the feedback

Algorithm 1 Simulation of Counterfactual Paths $\left\{Y_i^{T,*}, X_i^{T,*}\right\}$

Input: θ, $H(\lambda_i \mid \mathcal{I}_i^0)$, $\left\{f_t(X_{it} \mid \mathcal{I}_i^{t-1})\right\}_{t=1}^{T}$, alternative timing t_{0i}^*, and/or alternative initial conditions $\{Y_{i0}^*, X_{i0}^*\}$.
Set $\{Y_{i0}^*, X_{i0}^*\}$ to observed or chosen counterfactual initial conditions.
Draw $\lambda_i = (\alpha_i, \delta_{i0})'$ from $H(\lambda_i \mid \mathcal{I}_i^{0,*})$, with $\mathcal{I}_i^{0,*} = \{Y_{i0}^*, X_{i0}^*, t_{0i}^*\}$.
for $j = 1$ to J **do**
    Draw error term: $\varepsilon_{ij}^* \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.
    Update treatment effect: $\delta_{ij}^* = \rho_\delta \delta_{i,j-1}^* + \varepsilon_{ij}^*$.
end for
for $t = 1$ to T **do**
    Draw covariate: $X_{it}^* \sim f_t(\cdot \mid \mathcal{I}_i^{t-1,*})$.
    Draw error term: $U_{it}^* \sim \mathcal{N}(0, \sigma_U^2)$.
    Update outcome: $Y_{it}^* = \rho_Y Y_{i,t-1}^* + \alpha_i + (X_{it}^*)'\beta + \sum_{j \in \mathcal{J}} D_{it}^{j,*} \delta_{ij}^* + U_{it}^*$.
end for
Output: Counterfactual paths $\left\{Y_i^{T,*}, X_i^{T,*}\right\}$.

process $g(X_i^T \mid Y_i^T, \mathcal{I}_i^0)$ remains a separately identified component.

The homogeneous feedback assumption in (3) can be relaxed to allow for observed group-specific covariate adjustment rules. That is, let G_i denote an observed group indicator, such as industry, region, or demographic category, and assume that for each t, $f_t(X_{it} \mid \mathcal{I}_i^{t-1}, G_i, \lambda_i) = f_t(X_{it} \mid \mathcal{I}_i^{t-1}, G_i)$. That is, conditional on observable history *and* group membership, the covariate adjustment process does not depend on unobserved heterogeneity, but may vary across groups. Under this modification, the likelihood factorization applies group by group. In particular, for each g, θ and $H(\lambda_i \mid \mathcal{I}_i^0, G_i = g)$ are identified as before.

References

Bonhomme, S. (2025). Back to feedback: Dynamics and heterogeneity in panel data. Working Paper.

Botosaru, I. and Liu, L. (2025). Time-varying heterogeneous treatment effects in event studies. *arXiv preprint*. Discussion paper.

A Proof of Theorem 1

Proof. The proof proceeds by factoring the joint likelihood of the data to separate the feedback process for X_i^T from the structural outcome model for Y_i^T.

Let $\mathcal{L}(\lambda_i; Y_i^T, X_i^T \mid \mathcal{I}_i^0)$ denote the likelihood of the observed trajectory for unit i conditional on $\mathcal{I}_i^0$. By the law of total probability, the conditional likelihood of the observables given $\mathcal{I}_i^0$ is

$$f(Y_i^T, X_i^T \mid \mathcal{I}_i^0) = \int \mathcal{L}(\lambda_i; Y_i^T, X_i^T \mid \mathcal{I}_i^0) \mathrm{d}H(\lambda_i \mid \mathcal{I}_i^0). \tag{5}$$

Using Assumption 1, we decompose $\mathcal{L}(\lambda_i; Y_i^T, X_i^T \mid \mathcal{I}_i^0)$ as a product over time. For $t \geq 1$ the transition kernels depend on the history through $(\mathcal{I}_i^{t-1}, X_{it}, \lambda_i)$, and conditioning additionally on $\mathcal{I}_i^0$ does not change the transition densities. Thus we can write

$$\begin{aligned}\mathcal{L}(\lambda_i; Y_i^T, X_i^T \mid \mathcal{I}_i^0) &= \prod_{t=1}^T f(Y_{it} \mid \mathcal{I}_i^{t-1}, X_{it}, \lambda_i) f_t(X_{it} \mid \mathcal{I}_i^{t-1}, \lambda_i) \\ &= \underbrace{\left(\prod_{t=1}^T f(Y_{it} \mid \mathcal{I}_i^{t-1}, X_{it}, \lambda_i) \right)}_{= \mathcal{L}^{SE}(\lambda_i; Y_i^T \mid X_i^T, \mathcal{I}_i^0)} \times \underbrace{\left(\prod_{t=1}^T f_t(X_{it} \mid \mathcal{I}_i^{t-1}, \lambda_i) \right)}_{= g(X_i^T \mid Y_i^T, \mathcal{I}_i^0)}.\end{aligned}$$

By Assumption 2, the feedback term $g(X_i^T \mid Y_i^T, \mathcal{I}_i^0)$ defined in (4) does not depend on λ_i and is identified from the joint distribution $f(Y_i^T, X_i^T \mid \mathcal{I}_i^0)$ via standard factorization: for each $t = 1, \ldots, T$,

$$f_t(X_{it} \mid \mathcal{I}_i^{t-1}) = \frac{f(Y_i^{t-1}, X_i^t \mid \mathcal{I}_i^0)}{f(Y_i^{t-1}, X_i^{t-1} \mid \mathcal{I}_i^0)},$$

where the numerator and denominator are identified by marginalization, and the positivity condition in Assumption 2 ensures the ratio is well-defined.

Substituting (4) into (5) and rearranging, we define the reweighted conditional quasi-likelihood

$$\tilde{f}(Y_i^T \mid X_i^T, \mathcal{I}_i^0) = \frac{f(Y_i^T, X_i^T \mid \mathcal{I}_i^0)}{g(X_i^T \mid Y_i^T, \mathcal{I}_i^0)} = \int \mathcal{L}^{SE}(\lambda_i; Y_i^T \mid X_i^T, \mathcal{I}_i^0) \mathrm{d}H(\lambda_i \mid \mathcal{I}_i^0). \tag{6}$$

The right hand side of (6) matches the representation in Botosaru and Liu (2025), so applying their identification argument under the stated regularity conditions establishes the identification of θ and $H(\lambda_i \mid \mathcal{I}_i^0)$. Finally, $H(\lambda_i \mid t_{0i})$ and $H(\lambda_i)$ are identified by integrating $H(\lambda_i \mid \mathcal{I}_i^0)$ over the distribution of $\mathcal{I}_i^0$ conditional on t_{0i} and unconditionally, respectively. $\square$

B Supplemental Appendix

Lemma 1 (Elimination of Time Effects)**.** *Suppose the outcome equation* (1) *is augmented with additive time effects,*

$$Y_{it} = \rho_Y Y_{i,t-1} + \alpha_i + X_{it}'\beta + \sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} + \gamma_t + U_{it}, \quad t = 1, \dots, T.$$

Define the cross-sectional averages

$$\bar{Y}_t = \frac{1}{N}\sum_{i=1}^N Y_{it}, \quad \bar{X}_t = \frac{1}{N}\sum_{i=1}^N X_{it}, \quad \bar{\alpha} = \frac{1}{N}\sum_{i=1}^N \alpha_i, \quad \bar{U}_t = \frac{1}{N}\sum_{i=1}^N U_{it},$$

and

$$\bar{D}_t(\delta) = \frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij}.$$

Let the demeaned variables be

$$\dot{Y}_{it} = Y_{it} - \bar{Y}_t, \quad \dot{X}_{it} = X_{it} - \bar{X}_t, \quad \dot{\alpha}_i = \alpha_i - \bar{\alpha}, \quad \dot{U}_{it} = U_{it} - \bar{U}_t.$$

Then the demeaned outcome satisfies

$$\dot{Y}_{it} = \rho_Y \dot{Y}_{i,t-1} + \dot{\alpha}_i + \left(\sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} - \bar{D}_t(\delta) \right) + \dot{X}_{it}'\beta + \dot{U}_{it}, \tag{7}$$

so that the time effects $\{\gamma_t\}$ *are eliminated from the dynamic equation for* $\dot{Y}_{it}$*.*

Proof. Averaging the augmented outcome equation over $i = 1, \dots, N$ yields

$$\bar{Y}_t = \rho_Y \bar{Y}_{t-1} + \bar{\alpha} + \bar{D}_t(\delta) + \gamma_t + \bar{X}_t'\beta + \bar{U}_t.$$

Subtracting this average from the individual equation gives

$$\begin{aligned} Y_{it} - \bar{Y}_t = {} & \rho_Y (Y_{i,t-1} - \bar{Y}_{t-1}) + (\alpha_i - \bar{\alpha}) \\ & + \left(\sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} - \bar{D}_t(\delta) \right) + (\gamma_t - \gamma_t) + (X_{it} - \bar{X}_t)'\beta + (U_{it} - \bar{U}_t). \end{aligned}$$

The term $(\gamma_t - \gamma_t)$ is identically zero. Using the definitions of the demeaned variables, this simplifies to

$$\dot{Y}_{it} = \rho_Y \dot{Y}_{i,t-1} + \dot{\alpha}_i + \left(\sum_{j \in \mathcal{J}} D_{it}^j \delta_{ij} - \bar{D}_t(\delta) \right) + \dot{X}_{it}' \beta + \dot{U}_{it},$$

which is exactly (7). Hence the time effects $\{\gamma_t\}$ are eliminated from the dynamic equation for the demeaned outcome. □